\mathbf{\omega\_{opt} = R\_{y}^{-1}R\_{yx}}

\mathbf{\cong \ \hat{x} = \omega y}

\mathbf{[(G\_{n})\_{opt}^{H}]\_{:,k} = R\_{x\_{n}}^{-1}R\_{x\_{n}v\_{n}}}

\mathbf{[(G\_{n})\_{opt}^{H}]\_{:,k} = (H\_{n}U\_{n}U\_{n}^{H}H\_{n}^{H} + \rho I\_{n\_{R}})^{-1}[H\_{n}U\_{n}C\_{n}^{H}]\_{:,k}}

\mathbf{MSE\_{n,k} = LMMSE\_{n,k} = \sigma\_{w}^{2}[C\_{n}(H\_{n}U\_{n}U\_{n}^{H}H\_{n}^{H} + \rho I\_{n\_{R}})^{-1}C\_{n}^{H}]\_{k,k}}

\mathbf{LMMSE\_{n,k} = E[|\tilde{x}|^{2}] = E[|(\tilde{v}\_{n})\_{k}|^2]}

\mathbf{LMMSE\_{n,k} = [R\_{v\_{n}}]\_{k,k} - [R\_{v\_{n}x\_{n}}]\_{k,:}[(G\_{n})\_{opt}^{H}]\_{:,k}}

\mathbf{\because \ LMMSE = \sigma\_{x}^{2} - R\_{xy}\omega\_{opt}}

\mathbf{v\_{n} = a\_{n} + d\_{n}}

\mathbf{v\_{n} = (I\_{k} + B\_{n})b\_{n} = C\_{n}b\_{n}}

\mathbf{s\_{n} = U\_{n}b\_{n}}

\mathbf{x\_{n} = H\_{n}s\_{n} + w\_{n}}

\mathbf{y\_{n} = G\_{n}x\_{n}}

\mathbf{y\_{n} = G\_{n}H\_{n}U\_{n}b\_{n} + G\_{n}w\_{n}}

\mathbf{MSE\_{n,k} = E[|(y\_{n} - v\_{n})\_{k}|^{2}] }

\mathbf{ = E[(y\_{n} - C\_{n}b\_{n})(y\_{n} - C\_{n}b\_{n})^{H}]\_{k,k} }

\min\_{\bf C\_{n},U\_{n},G\_{n}} f\_{n}(MSE\_{n,1},...MSE\_{n,K})

s.t \ Tr({\bf U\_{n}U\_{n}^{H}}) = P\_{n} \ ; \ P\_{n}\geq 0

\min\_{\bf C\_{n},U\_{n},G\_{n}} \alpha\_{n} = f\_{n}(MSE\_{n,1},...MSE\_{n,K})

\displaystyle \sum\_{n=0}^{N-1}P\_{n} = KN \ ; \ P\_{n}\geq 0

\tilde{f}(P\_{0},P\_{1},....P\_{N-1}) = f(\hat{\alpha\_{0}}(P\_{0}),\hat{\alpha\_{1}}(P\_{1}),...,\hat{\alpha\_{N-1}}(P\_{N-1}))

\hat{\alpha\_{n}}(P\_{n})

E[{\bf b\_{n}}] = 0 ; E[{\bf b\_{n}b\_{n}^{H}}] = \sigma\_{a}^{2}{\bf I\_{K}}

E[{\bf w\_{n}}] = 0 ; E[{\bf w\_{n}w\_{n}^{H}}] = \sigma\_{w}^{2}{\bf I\_{n\_{R}}}

\rho = \sigma\_{w}^{2}/\sigma\_{a}^{2}

\mathbf{T(z) = S(z) - (A(z)-1)T(z) -bN}

MOD\_{M}(x) = x - 2\sqrt M \left \lfloor \displaystyle \frac{x+\sqrt M}{2\sqrt M} \right \rfloor

x = 1.99 \sqrt M ; MOD\_{M}(x) = -0.01 \sqrt M

x = 0.99 \sqrt M ; MOD\_{M}(x) = 0.99 \sqrt M

MOD\_{M}(x) \ \epsilon \ \{-\sqrt M,\sqrt M \ \}

\mathbf{MSE\_{n,k} = \sigma\_{w}^{2}([C\_{n}^{H}]\_{:,k})^{H}(H\_{n}U\_{n}U\_{n}^{H}H\_{n}^{H} + \rho I\_{n\_{R}})^{-1}[C\_{n}^{H}]\_{:,k}}

\mathbf{MSE\_{n,k} = \sigma\_{w}^{2}([C\_{n}^{H}]\_{:,k})^{H}L\_{n}L\_{n}^{H}[C\_{n}^{H}]\_{:,k} = \sigma\_{w}^{2}||[(C\_{n}L\_{n})^{H}]\_{:,k}||^{2}}

\mathbf{= \sigma\_{w}^{2} \displaystyle \sum\_{i=1}^{k-1}[L\_{n}]^{2}\_{i,i} \ |[(C\_{n}\tilde{L}\_{n})^{H}]\_{i,k}|^{2} \ +\sigma\_{w}^{2}[L\_{n}]^{2}\_{k,k}}

\mathbf{C\_{n}^{opt} = \tilde{L}\_{n}^{-1} = D\_{n}L\_{n}^{-1}}

MSE\_{n,k} = \sigma\_{w}^{2}{\bf [L\_{n}]\_{k,k}^{2}}

\mathbf{L\_{n} = \tilde{L}\_{n}D\_{n} \ ; \ D\_{n} = diag\{[L\_{n}]\_{1,1}, ..., [L\_{n}]\_{K,K}\}}

\mathbf{A = (X^{H}RX + \rho I\_{K})^{-1} = LL^{H}}

\mathbf{U\_{n} = X \ ; \ H\_{n}^{H}H\_{n} = R \ ; \ L\_{n} = L}

\min\_{\bf X} \phi({\bf d\_{L}^{2}(X)}) \\ st \ Tr({\bf XX^{H}}) = T

; \ T = P\_{n}

{\bf X\_{opt} = E\Gamma}

{\bf U\_{n}^{opt} = E\_{n}\Gamma\_{n}}

{\bf C\_{n} = I\_{K} \ ; \ B\_{n} = 0 \ \ \because L\_{n} \sim diag(..)}

f\_{n} ({MSE\_{n,k}}) = \displaystyle \prod\_{k=1}^{K} MSE\_{n,k}^{\beta\_{k}} \\ \\s.t. \ \ 0\leq \ \beta\_{1} \ \ \leq \ \ ..... \ \ \leq \ \beta\_{K}

{\bf X\_{opt} = E\Omega F} \\ s.t. \ {\bf\Omega = diag(\omega\_{1},...\omega\_{q})} \\ and \ \omega\_{i}^{2} = \left (\mu - \frac{\rho}{\eta\_{i}} \right)\_{+}

{\bf d\_{L}^{2}(X) = ([L]\_{1,1}^{2},....,[L]\_{q,q}^{2})}

{\ \displaystyle \sum\_{i=1}^{q}\omega\_{i}^{2} = T}

{\bf [d\_{L}^{2}(X\_{opt})]\_{k} = \left(\displaystyle \prod\_{i=1}^{q}\frac{1}{\eta\_{i} \omega\_{i}^{2} + \rho} \right)^{\frac{1}{q}}}

{ MSE\_{n} = \sigma\_{w}^{2} \left(\displaystyle \prod\_{i=1}^{q}\frac{1}{\eta\_{i} \omega\_{i}^{2} + \rho} \right)^{\frac{1}{q}}}

(\eta\_{1},...\eta\_{q})

\min\_{\bf C{n},G{n},U{n}} \ f({MSE\_{n,k}}) = \displaystyle \sum\_{n=0}^{N-1} \sum\_{k=1}^{K} MSE\_{n,k} \\ \center s.t. \ \ \displaystyle \sum\_{n=0}^{N-1}Tr(U\_{n}U\_{n}^{H}) = KN \\ \center where \ f(\alpha\_{0},..., \alpha\_{N-1}) = \displaystyle \sum\_{n=0}^{N-1}\alpha\_{n} \\ \center \alpha\_{n} = f\_{n}(MSE\_{n,1},..., MSE\_{n,K}) = \displaystyle \sum\_{k=1}^{K} MSE\_{n,k}

\hat{\alpha\_{n}}(P\_{n}) = K(MSE\_{n,k})

\center {\bf \lambda\_{A}(X)} = (\lambda\_{1},..., \lambda\_{q}) \\ \center {\bf d\_{L}^{2}(X)} <\_{\pi} {\bf \lambda\_{A}(X)} \Rightarrow \phi({\bf \lambda\_{A}(X)}) \leq\phi({\bf d\_{L}^{2}(X)})

{\bf \tilde{X} = XQ} \ s.t \ Q : unitary \\ and \ \ {\bf \tilde{X}^{H}RX} \ is \ diag

\phi({\bf d\_{L}^{2}(\tilde{X})}) = \phi({\bf \lambda\_{A}(X)}) \leq\phi({\bf d\_{L}^{2}(X)})

{\bf \tilde{X}} \ s.t. \ {\bf \tilde{X}^{H} R \tilde{X} = X^{H} RX} \\ {\bf \tilde{X} = E\Sigma} \ and \ Tr({\bf \tilde{X}\tilde{X}^{H}}) \leq Tr({\bf XX^{H}})

{\bf X\_{opt} = \hat{X} = \alpha \tilde{X} = E(\alpha \Sigma) = E\Gamma} \\ where \ \alpha = \sqrt{\frac{T}{Tr({\bf \tilde{X}\tilde{X}^{H}})}} \ \geq 1

{\bf x\_{GM} \prec\_{\Pi} x} \ where \ [{\bf x\_{GM}}]\_{i} = \left(\prod\_{j=1}^{q}x\_{j} \right )^{\frac{1}{q}}

{\bf F} \ : \ unitary \ s.t. \ {\bf F^{H} A F = LL^{H}} \\ {\bf [L]\_{k,k}} \ are \ all \ equal

(X^{H}RX + \rho I\_{K}})^{-1}

\phi(\Delta^{1/q},...,\Delta^{1/q}) = \phi({\bf d\_{L}^{2}(XF)}) \leq \phi({\bf d\_{L}^{2}(X)})

find \ {\bf \tilde{X}} \ which \ is \ the \ optimum \ of \\ \displaystyle \min\_{\bf X} \ \Delta = det(({\bf X^{H}RX + \rho I\_{K}})^{-1}) \\ \\ s.t. \ Tr({\bf XX^{H}}) = T

{\bf \hat{X} = X\_{opt} = \tilde{X}F}

{\bf y = GHUb + Gw} \ where \ {\bf H = diag(H\_{0},...H\_{N-1})}